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Noncommutative Tachyons And String Field Theory

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It has been shown recently that by turning on a large noncommutativity parameter, the description of tachyon condensation in string theory can be drastically simplified. We reconsider these issues from the standpoint of string field theory, showing that, from this point of view, the key fact is that in the limit of a large B-field, the string field algebra factors as the product of an algebra that acts on the string center of mass only and an algebra that acts on all other degrees of freedom carried by the string.

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Recently, solitons in scalar field theories on noncommutative spacetimes with very large noncommutativity parameter θ have been constructed in a strikingly simple way [1]. This insight has been applied [2,3] to string theory with strong noncommutativity to test the predictions about tachyon condensation and brane annihilation [4].

The purpose of the present paper is to examine these issues in the context of open string field theory, as formulated in the language of noncommutative geometry [5]. String field theory has been used to explore tachyon condensation [6-12], generally at $\theta = 0$. As one might expect [13,14], there is considerable simplification for large θ, and so we will in this paper consider in string field theory language the solutions studied in [1-3]. We will carry out this discussion in flat $\mathbf{R}^{26}$ or $\mathbf{R}^{10}$.

Factorization Of The Algebra

In string field theory, the starting point is an associative algebra $\mathcal{A}$ that is built by multiplying string fields. The naive idea for constructing such an algebra is to start with the association of open string states with vertex operators $\mathcal{V}$. An open string vertex operator is of course inserted at the boundary of the open string, at some proper time τ along the boundary. Naively speaking, to multiply string states we would like to just multiply the corresponding vertex operators. The trouble is that the product $\mathcal{V}\cdot\mathcal{V}'$ of open string vertex operators at the same point on the boundary is not well-defined, because of the familiar short distance singularities of products of local quantum field operators.

We do have an operator product expansion (OPE)

$$\mathcal{V}(\tau)\mathcal{V}'(\tau') \to \sum_k c_k|\tau-\tau'|^{-a_k}\mathcal{V}_k(\tau') \quad \text{for } \tau\to\tau'. \tag{1}$$

The coefficients c_k in this expansion depend on whether $\tau > \tau'$ or $\tau < \tau'$; this is the origin of noncommutativity. There does not seem to be any elegant way to eliminate the τ dependence and extract an associative algebra $\mathcal{A}$ from the operator product expansion. In open string field theory, this is done by making rather special choices of local parameters for insertions of vertex operators; the construction is perhaps most naturally described in terms of gluing of open string states [5]. At any rate, in this paper, we will only need properties of the algebra $\mathcal{A}$ that follow in a very general way from the properties of the operator product algebra. Technical details in the definition of $\mathcal{A}$ will not be important.

The operator product expansion conserves the "ghost number" of the vertex operators, and hence $\mathcal{A}$ is graded by ghost number. The classical string field A is a ghost number

one element of $\mathcal{A}$. The worldsheet BRST operator Q is a ghost number one derivation of the algebra (that is, $Q(A*A') = QA*A' + (-1)^A A*QA'$), and the equation of motion of bosonic open string field theory is

$$QA + A*A = 0. \tag{2}$$

A similar construction can be made for open superstrings, but has been argued to have technical difficulties associated with meeting of picture-changing operators on the world-sheet [15]. A modification has been proposed to circumvent this difficulty [16]. The effect of this is to replace (2) by a nonpolynomial equation which, for our purposes in the present paper, can be treated in precisely the same way.

Now, the OPE algebra of open string vertex operators has a subalgebra $\mathcal{A}_0$ in which one does not use the string center of mass coordinate. Thus, $\mathcal{A}_0$ contains vertex operators that depend in an arbitrary fashion on the ghosts b and c and the derivatives of the space-time coordinates X^i, $i = 1, \ldots, 26$, all taken at zero spacetime momentum. $\mathcal{A}_0$ contains, for example, $b\partial c(\partial X^1)^{22}\partial^3 X^2$, but not $b\partial c(\partial X^1)^{22}\partial^3 X^2 e^{ip\cdot X}$ with $p \neq 0$. Operators of zero momentum are closed under OPE's, and so $\mathcal{A}_0$ is a subalgebra of $\mathcal{A}$.

One may ask whether there is a complementary subalgebra $\mathcal{A}_1$ generated *only* by the $e^{ip\cdot X}$ and without ∂X, $\partial^2 X$, etc. Normally, the answer to this question is "no," since even if one merely makes a classical Taylor series expansion, the operator products of exponentials involve also the derivatives of X:

$$e^{ip\cdot X}(\tau)e^{iq\cdot X}(\tau') \to e^{i(p+q)\cdot X}(\tau') + i(\tau-\tau')p\cdot\partial X e^{i(p+q)\cdot X}(\tau') + \ldots. \tag{3}$$

However, there is a limit in which one actually can factorize $\mathcal{A}$ in terms of commuting subalgebras as $\mathcal{A} = \mathcal{A}_0 \otimes \mathcal{A}_1$, where as suggested above $\mathcal{A}_0$ consists of vertex operators of zero momentum and $\mathcal{A}_1$ is generated by operators $e^{ip\cdot X}$. This is the limit in which the NS two-form field B is constant and large [13,14]. In fact, we assume that B is of maximal rank in 10- or 26-dimensional Euclidean space.

To see this factorization, we first recall the form of the worldsheet propagator in the presence of a B-field. We take the string world-sheet to consist of the upper half plane. The propagator between boundary points τ, τ' on the real axis is then [17,18]

$$\langle X^i(\tau)X^j(\tau') = -\alpha'\left(\frac{1}{g+2\pi\alpha' B}\right)_S^{ij}\ln(\tau-\tau')^2 + i\pi\alpha'\left(\frac{1}{g+2\pi\alpha' B}\right)_A^{ij}\epsilon(\tau-\tau'). \tag{4}$$

Here g is the closed string metric and $(\ )_S$, $(\ )_A$ denote the symmetric and antisymmetric part of a matrix.

Now we take the limit $B \to \infty$ with g, α' fixed. To be definite, take $B = tB_0$ with $t \to \infty$. (This is the same limit as in [14], but parametrized differently.) If we set

$$X^i = Y^i/\sqrt{t}, \tag{5}$$

then the propagator becomes

$$\langle Y^i(\tau)Y^j(\tau')\rangle = \frac{1}{t(2\pi)^2\alpha'}\left(\theta^2\right)^{ij}\ln(\tau-\tau')^2 + \frac{i}{2}\theta^{ij}\epsilon(\tau-\tau'), \tag{6}$$

where $\theta = 1/B_0$.

Now for $t \to \infty$, the $e^{iq\cdot Y}$ do generate a closed algebra as the $\ln(\tau-\tau')^2$ term does not contribute. (We will make this more explicit momentarily.) This is the center of mass algebra $\mathcal{A}_1$. What about $\mathcal{A}_0$, the algebra that *doesn't* contain the center of mass momentum? As $\partial Y(\tau)\cdot\partial Y(\tau') \sim t^{-1}/(\tau-\tau')^2$, we see that if we use $\sqrt{t}\,\partial^n Y$ as the algebra generators of $\mathcal{A}_0$, then the structure constants are independent of t. A typical element of $\mathcal{A}_0$ is then

$$b\partial c\cdot\sqrt{t}\,\partial^{n_1}Y^{a_1}\cdot\sqrt{t}\,\partial^{n_2}Y^{a_2}\cdot\sqrt{t}\,\partial^{n_3}Y^{a_3}, \tag{7}$$

with a factor of $\sqrt{t}$ for each $\partial^n Y$.

So in this limit, we have two algebras $\mathcal{A}_0$ and $\mathcal{A}_1$. They commute and the full algebra is $\mathcal{A} = \mathcal{A}_0 \otimes \mathcal{A}_1$. To verify that $\mathcal{A}_0$ and $\mathcal{A}_1$ commute, we simply have to observe that

$$\sqrt{t}\,\partial^n Y(\tau)e^{iq\cdot Y}(\tau') \sim \frac{1}{\sqrt{t}}(\tau-\tau')^{-n}e^{iq\cdot Y}, \tag{8}$$

since the only term in the propagator that contributes is the logarithmic term, proportional to t^{-1}. The right hand side vanishes for $t \to \infty$. Finally, to verify that $\mathcal{A}_1$ is closed under OPE's, we note that when we expand

$$e^{ip\cdot Y}(\tau)e^{iq\cdot Y}(\tau') \sim e^{-\frac{1}{2}i\theta_{ij}p^iq^j}e^{i(p+q)\cdot Y}(\tau')(1 + i(\tau-\tau')p\cdot\partial Y + \ldots), \tag{9}$$

the corrections proportional to ∂Y can be dropped since the factor of ∂Y is not accompanied by a factor of $\sqrt{t}$. The OPE algebra $\mathcal{A}_1$ thus reduces to

$$e^{ip\cdot Y}(\tau)e^{iq\cdot Y}(\tau') \sim e^{-\frac{1}{2}i\theta_{ij}p^iq^j}e^{i(p+q)\cdot Y}(\tau') \tag{10}$$

which is the algebra of functions on noncommutative $\mathbf{R}^{10}$ or $\mathbf{R}^{26}$; as explained in [14], the usual complications of the OPE disappear, because the dimensions vanish and the right hand side has no dependence on $\tau - \tau'$.

So far we have assumed that the B-field has maximal rank. If B has less than maximal rank, we modify the factorization so that $\mathcal{A}_1$ is generated by $e^{ip\cdot X}$ where p is tangent to the noncommutative directions and $\mathcal{A}_0$ is generated by all other operators. We still get a factorization $\mathcal{A} = \mathcal{A}_0 \otimes \mathcal{A}_1$ in terms of commuting subalgebras. In this more general case, an operator in $\mathcal{A}_0$ may carry momentum, but in commutative directions only, while $\mathcal{A}_1$, on the other hand, will be the algebra of functions in the noncommutative directions. $\mathcal{A}_1$ is a down-to-earth algebra that can be described concretely in finite-dimensional terms, while $\mathcal{A}_0$ contains all of the mysterious stringy complications.

By a similar scaling, the BRST operator Q acts on $\mathcal{A}_0$ and commutes with $\mathcal{A}_1$. So the string field equation $0 = QA + A * A$ makes sense for $A \in \mathcal{A}_0$.

Tachyon Condensation

For bosonic strings, there are at least two important solutions known with $A \in \mathcal{A}_0$. One of them is $A = 0$ and describes the ordinary open string vacuum. The other, which will here be called A_0, was first explored numerically in [6] and is now understood to describe tachyon condensation to "nothing," that is, to a state with only closed strings. This means, in particular, that the corresponding A_0-A_0 open strings, with boundary conditions at both ends determined by the classical open string solution A_0, have no physical excitations.

Now more generally, we could introduce 2×2 Chan-Paton factors and start with two $D25$-branes. The solution

$$A = \begin{pmatrix} 0 & 0 \\ 0 & A_0 \end{pmatrix} \tag{11}$$

describes annihilation to a state with just one $D25$-brane. There are now several kinds of open strings:

(1) The 0-0 open strings describe physical excitations of the surviving $D25$-brane.

(2) The 0-A_0 and A_0-A_0 open strings are expected to have no physical excitations.

Since the solution A_0 lies in $\mathcal{A}_0$, which (after suitably rescaling the coordinates) is completely independent of B, the solution A_0 is completely insensitive to the B-field. Now, let us specialize to the limit of large B and invoke the idea of [1]. Let $\rho \in \mathcal{A}_1$ be any projection operator, that is any element with

$$\rho^2 = \rho. \tag{12}$$

Then as $[Q,\rho]=0$, we see that $A=A_0\otimes\rho$ obeys $0=QA+A*A$ if A_0 does. Equivalently, since (12) implies that $(1-\rho)^2=(1-\rho)$, we can solve the equation with

$$A=A_0\otimes(1-\rho). \tag{13}$$

Now, following [1], represent $\mathcal{A}_1$ as the algebra of operators on a Hilbert space $\mathcal{H}$. The endpoint of a string has a Chan-Paton label that takes values in $\mathcal{H}$. Take ρ to be the projector onto a finite-dimensional subspace of $\mathcal{H}$, say a subspace V of dimension n. Write $\mathcal{H}=V\oplus W$ where W is the orthocomplement of V; so $\rho|_V=1$ and $\rho|_W=0$. In expanding around the solution $A=A_0\otimes(1-\rho)$, we get several kinds of open strings:

$(1)'$ The V-V open strings, that is strings each of whose endpoints are labeled by vectors in V, are governed by the usual equations of open string theory except that the effective open string algebra for these strings is $\mathcal{A}_0\otimes M_n$, where M_n is the algebra of $n\times n$ complex matrices acting on V. Hence the momentum of these strings is always tangent to the commutative directions in spacetime, and there are effective $n\times n$ Chan-Paton factors. These modes describe the physical excitations of n $D(25-2p)$-branes, where $2p$ is the number of noncommutative directions.

$(2)'$ The V-W and W-W open strings are governed by the same equations that describe the 0-A_0 and A_0-A_0 open strings discussed above; so if the usual conjectures about tachyon condensation are true, then these open strings have no physical excitations.

Thus, this solution describes annihilation of a $D25$-brane to a collection of n parallel $D(25-2p)$-branes, for arbitrary p and n.

Type IIA

At the very formal level of our discussion, we can consider tachyon condensation for unstable $D9$-branes of Type IIA in much the same way. In factorizing $\mathcal{A}=\mathcal{A}_0\otimes\mathcal{A}_1$, we include the superconformal ghosts and worldsheet fermions in $\mathcal{A}_0$; $\mathcal{A}_1$ is as in the case of the bosonic string the algebra of functions in the noncommuting directions of spacetime. There is, conjecturally, still a solution A_0 that describes tachyon condensation, and a more general solution $A=A_0\otimes(1-\rho)$ which, if ρ is the projection operator to an n-dimensional subspace, describes annihilation to a system of n $D(9-2p)$-branes. As noted in [2,3], a further subtlety arises because for open string excitations of a Type IIA $D9$-brane, there is a $\mathbf{Z}_2$ symmetry that changes the sign of the tachyon field. Let A_0' be the conjugate solution with opposite tachyon field. The solutions A_0 and A_0' describe tachyon condensation to

two different closed string vacua that differ by the sign of the tachyon field; in fact, they differ by one unit of the Ramond-Ramond zero-form $G_0/2\pi$ [19].

If ρ_1, ρ_2 obey $\rho_1^2 = \rho_1$, $\rho_2^2 = \rho_2$, $\rho_1\rho_2 = \rho_2\rho_1 = 0$, we can make a more general solution with $A = A_0 \otimes \rho_1 + A_0' \otimes \rho_2$. A special case of this with $\rho_1 = \rho$, $\rho_2 = 1 - \rho_1$ is

$$A = A_0 \otimes (1 - \rho) + A_1 \otimes \rho. \tag{14}$$

Let us consider ρ to be the projector onto all quantum states that are supported within a large region Ω in the noncommutative phase space. This is only a rough, semiclassical description of ρ, but it should be good if Ω is large enough. (14) describes tachyon condensation to a state in which the tachyon field has one sign outside of Ω and another sign inside Ω; because the two states differ by one unit of $G_0/2\pi$, there is a supersymmetric $D8$-brane wrapped on the boundary of Ω. As noted in [2,3], it is perplexing that in this approximation the tension of the $D8$-brane appears to be zero.

To explore this puzzle in a little more detail (but without claiming to resolve it), consider the case of two noncommutative directions with coordinates x, y obeying $[x, y] = -i\theta$. Suppose that Ω is a disc and that we want ρ to be a projector onto an n-dimensional subspace. Then the area of Ω should be $A = 2\pi\theta n$, so its radius is $r = \sqrt{2\theta n}$. ρ is approximately 1 deep inside Ω and approximately 0 far from Ω; the scale of variation of ρ is approximately the same as the width in space of the outermost quantum state onto which ρ projects. (If we try to make ρ vary more slowly than that, there will be states on which it is not equal approximately to either 0 or 1.) That outermost state fills a cylindrical shell near the boundary of Ω of area $2\pi\theta$; the radial thickness of the shell is thus $\Delta r = \theta/r = \sqrt{\theta/2n}$. The validity of our description rests on neglecting the logarithmic term in (6), which is proportional to $\theta^2/t\alpha'$; this term can be considered small if the scale of variation of the solution is large compared to $\theta/\sqrt{t\alpha'}$. The condition we need is thus $\Delta r >> \theta/\sqrt{t\alpha'}$ or

$$\frac{A}{t} << \alpha'. \tag{15}$$

Since A/t is the area of Ω in the original coordinates X, before the rescaling (5), this means that the solution (14) is actually only valid if the area of Ω in string units is much less than one.

Type IIB

Now what can we say about tachyon condensation for Type IIB superstrings? For Type IIB, we could first of all consider a $D9$- or $D\overline{9}$-brane with $A=0$. The corresponding boundary conditions for open strings we will call 0 and $\overline{0}$, respectively.

The combined $D9$-$D\overline{9}$ system is believed to admit a somewhat more interesting solution. First of all, to describe a $D9$-$D\overline{9}$ system, we use 2×2 Chan-Paton matrices, but with the opposite GSO projection for the off-diagonal terms. Thus the string field takes the form

$$A=\begin{pmatrix} B & T \\ \overline{T} & B' \end{pmatrix}, \tag{16}$$

where B and B' have the usual GSO projections and T and $\overline{T}$ have the opposite ones; thus B and B' describe gauge fields as well as stringy excitations, while T and $\overline{T}$ have a tachyon at the lowest level. There is a symmetry

$$T\to e^{-i\theta}T,\quad \overline{T}\to e^{i\theta}\overline{T}. \tag{17}$$

It is believed that there exists a solution that describes tachyon condensation to the closed string vacuum. It has been explored numerically [9,11,12] but is not known in closed form; we merely write it as

$$A_0=\begin{pmatrix} \alpha & \beta \\ \overline{\beta} & \gamma \end{pmatrix} \tag{18}$$

Because of the symmetry (17), it can be generalized to

$$A_\theta=\begin{pmatrix} \alpha & e^{-i\theta}\beta \\ e^{i\theta}\overline{\beta} & \gamma \end{pmatrix}. \tag{19}$$

As in the discussion of the bosonic string, we can add extra uncondensed $D9$'s and $D\overline{9}$'s and mix the two solutions. Thus, in a larger space, we can consider the string field

$$A=\begin{pmatrix} 0 & 0 \\ 0 & A_\theta \end{pmatrix}, \tag{20}$$

where the upper left block describes excitations of a $D9$ or $D\overline{9}$. This field certainly obeys the equations of string field theory, and describes partial annihilation of a system of $D9$'s and $D\overline{9}$'s, leaving a single brane. The open strings in expanding around this solution can be classified as 0-0 open strings, which describe ordinary open string excitations, and 0-θ, θ-0, and θ-θ open strings, all of which describe no physical modes at all if the usual hypotheses about tachyon condensation are correct.

In the large B limit, the string field algebra factors as $\mathcal{A} = \mathcal{A}_0 \otimes \mathcal{A}_1$ just as before. We want to generalize the solution $A_\theta \in \mathcal{A}_0$ to include the $\mathcal{A}_1$ factor, by a suitable generalization of the previous ansatz $A = A_0 \otimes \rho$.

For this, we let σ be an element of $\mathcal{A}_1$ and $\overline{\sigma}$ its complex conjugate (or hermitian adjoint in a Hilbert space representation). We take

$$A = \begin{pmatrix} \alpha \otimes \overline{\sigma}\sigma & \beta \otimes \overline{\sigma} \\ \overline{\beta} \otimes \sigma & \gamma \otimes \sigma\overline{\sigma} \end{pmatrix}. \tag{21}$$

To obey $QA + A * A = 0$ given that A_0 obeys this equation, [1] the properties we need are

$$\sigma\overline{\sigma}\sigma = \sigma, \quad \overline{\sigma}\sigma\overline{\sigma} = \overline{\sigma}. \tag{22}$$

If σ is invertible, these equations say that σ is unitary. On an eigenspace with $\sigma = e^{i\theta}$, we get

$$A = A_\theta = \begin{pmatrix} \alpha & e^{-i\theta}\beta \\ e^{i\theta}\beta & \gamma \end{pmatrix}. \tag{23}$$

This solution describes tachyon condensation to a state with no physical excitations.

The fun comes when σ is not invertible. Let V be the kernel of σ, and W the kernel of $\overline{\sigma}$ (or equivalently the cokernel of σ). Let n and m be the dimensions of V and W, and M_n, M_m the algebras of matrices acting on V and W respectively; we suppose that n and m are finite. The equations (22) mean in general that σ is a unitary isomorphism between the orthocomplement of V and the orthocomplement of W.

If $2p$ is the number of noncommutative directions, then string states whose endpoints are labeled by vectors in V describe the excitations of n $D(9-2p)$-branes, and those whose endpoints are labeled by vectors in W describe the excitations of m $D(\overline{9-2p})$-branes. The V-W open strings are equivalent to conventional $D(9-2p)$-$D(\overline{9-2p})$ open strings. Other open string excitations of this system are governed by the same equations as the 0-θ and θ-θ excitations of the the solution (20), and describe no physical excitations at all, if the conventional hypotheses are correct. This solution thus describes tachyon condensation down to a system with n $D(9-2p)$-branes and m $D(\overline{9-2p})$-branes. The net $D(9-2p)$-brane charge is $n-m$, which is the same as the index of the operator σ.[2]

[1] And similarly for any other equation, like the one in [16], that is constructed by multiplication of string fields as well as operations like Q that commute with $\mathcal{A}_1$.

[2] This solution has been described in a more detailed setting in section 4 of [3]. Note in eqn. (4.8) of that paper an operator of index 1.

To describe explicitly a solution of (22) with nonzero index, suppose that there are two noncommutative directions with coordinates x, y, with

$$[x, y] = -i\theta, \quad \theta > 0. \tag{24}$$

We introduce the creation and annihilation operators

$$a = \frac{x - iy}{\sqrt{2\theta}}, \quad \overline{a} = \frac{x + iy}{\sqrt{2\theta}}, \quad [a, \overline{a}] = 1. \tag{25}$$

a and $\overline{a}$ are represented on a Hilbert space $\mathcal{H}$ that contains a vector $|0\rangle$ with $a|0\rangle = 0$. The kernel of a is generated by $|0\rangle$, and $\overline{a}$ has no kernel. We let

$$\begin{aligned} \sigma &= \frac{1}{\sqrt{\overline{a}a + 1}} a \\ \overline{\sigma} &= \overline{a} \frac{1}{\sqrt{\overline{a}a + 1}}. \end{aligned} \tag{26}$$

Clearly, the kernel of σ is generated by $|0\rangle$, and the kernel of $\overline{\sigma}$ is empty; so the index of σ is 1. A short computation shows that $\sigma\overline{\sigma} = 1$, which implies (22). From (25), we have

$$\sigma = \frac{1}{\sqrt{x^2 + y^2 + \theta}} (x - iy). \tag{27}$$

If σ is treated as a classical function of x and y, then for $x, y \to \infty$ we have $|\sigma| = 1$. Ignoring the commutator $[x, y]$ is valid in describing the behavior near infinity. We can thus regard σ near infinity as a $U(1)$-valued function on a circle; as such, its winding number is -1. Thus, in this particular case, the index equals minus the winding number. This relation is a special case of the general Atiyah-Singer index theorem. Since the index and the winding number are both topological invariants, the relation between the index and the winding number can be proved, in this particular problem, by computing the index for one operator of every possible index, for example $\sigma = (1/\sqrt{a^n \overline{a}^n}) a^n$ for positive index or $\sigma = \overline{a}^n (1/\sqrt{a^n \overline{a}^n})$ for negative index. We leave details to the reader. The identification of the D-brane charge with the winding number of the tachyon field was the original proposal in [4].

Electric Flux Tubes?

In this paper, we have formulated in a slightly more abstract language many solutions that were described in [2,3]. A notable exception is the solution describing fundamental strings as electric flux tubes that is proposed in section 5 of [3]. This solution appears to

depend on properties of the string theory effective action that are more specific than the general features that have been exploited in the present paper.

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